DSC- 1C REAL ANALYSIS BS: 304

Theory: 4 credits and Practical 1 credit
Theory: 4 hours/week and Practicals : 2 hours/ week

Objective : The course is aimed at exposing the students to the foundations of analysis which will be useful in understanding various physical phenomena.

Outcome: After the completion of the course students will be in a position to appreciate beauty and applicability of the course.

Unit- I

Sequences- Limits of sequences- A Discussion about Proofs- Limit Theorems for Sequences – Monotone Sequences and Cauchy Sequences

Unit- II

Subsequences- Lim sup's and Lim inf's Series- Alternating Series and Integrals Tests. Continuity : Continuous functions- Properties of Continuous functions.

Unit – III

Sequence and Series of Functions: Power Series- Uniform Convergence – More on Uniform Convergence- Differentiation and Integration of Power Series (Theorems in this section without Proofs)

Unit – IV

Integration : The Riemann Integral- Properties of Riemann Integral- Fundamental Theorem of Calculus.

Text : Kenneth A Ross, Elementary Analysis- The Theory of Calculus

References :

William F.Trench: Introduction to Real Analysis

Lee Larson: Introduction to Real Analysis

Shanti Narayan and Mittal: Mathematical Analysis

Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner: Elementary Real Analysis

Sudhir R. Ghorpade Balmohan V. Limaye: A Course in Calculus and Real Analysis

2.5.1 Practicals Question Bank

Real Analysis

Unit-I

1. For each sequence below, determine whether it converges and, if it converges, give its limit. No proofs are required.

   (a) $a_n = \frac{n}{n+1}$

   (b) $b_n = \frac{n^2+3}{n^2-3}$

   (c) $c_n = 2^{-n}$

   (d) $t_n = 1 + \frac{2}{n}$

   (e) $x_n = 73 + (-1)^n$

   (f) $s_n = (2)^{\frac{1}{n}}$

2. Determine the limits of the following sequences, and then prove your claims.

   (a) $a_n = \frac{n}{n^2+1}$

   (b) $b_n = \frac{7n-19}{3n+7}$

   (c) $c_n = \frac{4n+3}{7n-5}$

   (d) $d_n = \frac{2n+4}{5n+2}$

   (e) $s_n = \frac{1}{n} \sin n$

3. Suppose $\lim a_n = a$, $\lim b_n = b$, and $s_n = \frac{a_n^3+4a_n}{b_n^2+1}$. Prove $\lim s_n = \frac{a^3+4a}{b^2+1}$ carefully, using the limit theorems.

4. Let $x_1 = 1$ and $x_{n+1} = 3x_n^2$ for $n \geq 1$.

   (a) Show if $a = \lim x_n$, then $a = \frac{1}{3}$ or $a = 0$.

   (b) Does $\lim x_n$ exist? Explain.

   (c) Discuss the apparent contradiction between parts (a) and (b).

5. Which of the following sequences are increasing? decreasing? bounded?

   (a) $\frac{1}{n}$

   (b) $\frac{(-1)^n}{n^2}$

   (c) n^5

   (d) $\sin(\frac{n\pi}{7})$

   (e) $(-2)^n$

   (f) $\frac{n}{3^n}$

6. Let (s_n) be a sequence such that $|s_{n+1} - s_n| < 2^{-n}$ for all $n \in \mathbb{N}$. Prove (s_n) is a Cauchy sequence and hence a convergent sequence.

7. Let (s_n) be an increasing sequence of positive numbers and define $\sigma_n = \frac{1}{n}(s_1 + s_2 + ... + s_n)$. Prove (σ_n) is an increasing sequence.

8. Let $t_1 = 1$ and $t_{n+1} = [1 - \frac{1}{4n^2}].t_n$ for $n \geq 1$.

   (a) Show $\lim t_n$ exists.

   (b) What do you think $\lim t_n$ is?

9. Let $t_1 = 1$ and $t_{n+1} = [1 - \frac{1}{(n+1)^2}].t_n$ for all $n \geq 1$.

   (a) Show $\lim t_n$ exists.

   (b) What do you think $\lim t_n$ is?

   (c) Use induction to show $t_n = \frac{n+1}{2n}$.

   (d) Repeat part (b).

10. Let $s_1 = 1$ and $s_{n+1} = \frac{1}{3}(s_n + 1)$ for $n \geq 1$.

   (a) Find s_2, s_3 and s_4.

   (b) Use induction to show $s_n > \frac{1}{2}$ for all n.

   (c) Show (s_n) is a decreasing sequence.

   (d) Show $\lim s_n$ exists and find $\lim s_n$.

Unit-II

11. Let $a_n = 3 + 2(-1)^n$ for $n \in \mathbb{N}$.

   (a) List the first eight terms of the sequence (a_n).

   (b) Give a subsequence that is constant [takes a single value].
   Specify the selection function σ.

12. Consider the sequences defined as follows:

   $a_n = (-1)^n$, $b_n = \frac{1}{n}$, $c_n = n^2$, $d_n = \frac{6n+4}{7n-3}$.

   (a) For each sequence, give an example of a monotone subsequence.

   (b) For each sequence, give its set of subsequential limits.

   (c) For each sequence, give its lim sup and lim inf.

   (d) Which of the sequences converges? diverges to $+\infty$? diverges to $-\infty$?

   (e) Which of the sequences is bounded?

13. Prove $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.

14. Let (s_n) and (t_n) be the following sequences that repeat in cycles of four:

$$(s_n) = (0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, ...)$$

$$(t_n) = (2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, ...)$$

   Find

   (a) $\liminf s_n + \liminf t_n$,

   (b) $\liminf(s_n + t_n)$,

   (c) $\liminf s_n + \limsup t_n$,

   (d) $\limsup(s_n + t_n)$,

(e) $\limsup s_n + \limsup t_n$, **(f)** $\liminf(s_n t_n)$,

(g) $\limsup(s_n t_n)$.

15. Determine which of the following series converge. Justify your answers.

(a) $\sum \frac{n^4}{2^n}$ **(b)** $\sum \frac{2^n}{n!}$

(c) $\sum \frac{n^2}{3^n}$ **(d)** $\sum \frac{n!}{n^4+3}$

(e) $\sum \frac{\cos^2 n}{n^2}$ **(f)** $\sum_{n=2}^{\infty} \frac{1}{\log n}$

16. Prove that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges.

17. Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges.
Hint: Show $\sqrt{a_n b_n} \leq a_n + b_n$ for all n.

18. We have seen that it is often a lot harder to find the value of an infinite sum than to show it exists. Here are some sums that can be handled.

(a) Calculate $\sum_{n=1}^{\infty}(\frac{2}{3})^n$ and $\sum_{n=1}^{\infty}(-\frac{2}{3})^n$.
(b) Prove $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. Hint: Note that $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n}[\frac{1}{k} - \frac{1}{k+1}]$.
(c) Prove $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. Hint: Note $\frac{k-1}{2^{k+1}} = \frac{k}{2^k} - \frac{k+1}{2^{k+1}}$.
(d) Use (c) to calculate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

19. Determine which of the following series converge. Justify your answers.

(a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\log n}$ **(b)** $\sum_{n=2}^{\infty} \frac{\log n}{n}$

(c) $\sum_{n=4}^{\infty} \frac{1}{n(\log n)(\log\log n)}$ **(d)** $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$

20. Show $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if and only if $p > 1$.

UNIT-III

21. For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

(a) $\sum n^2 x^n$ **(b)** $\sum (\frac{x}{n})^n$

(c) $\sum (\frac{2^n}{n^2}) x^n$ **(d)** $\sum (\frac{n^3}{3^n}) x^n$

(e) $\sum (\frac{2^n}{n!}) x^n$ **(f)** $\sum \left(\frac{1}{(n+1)^2 2^n}\right) x^n$

(g) $\sum(\frac{3^n}{n.4^n})x^n$ **(h)** $\sum\big(\frac{(-1)^n}{n^2.4^n}\big)x^n$

22. For $n = 0, 1, 2, 3, ...,$ let $a_n = \big[\frac{4+2(-1)^n}{5}\big]^n$.

    (a) Find $\limsup(a_n)^{1/n}$, $\liminf(a_n)^{1/n}$, $\limsup|\frac{a_{n+1}}{a_n}|$ and $\liminf|\frac{a_{n+1}}{a_n}|$.

    (b) Do the series $\sum a_n$ and $\sum(-1)^n a_n$ converge? Explain briefly.

23. Let $f_n(x) = \frac{1+2\cos^2 nx}{\sqrt{n}}$. Prove carefully that (f_n) converges uniformly to 0 on $\mathbb{R}$.

24. Prove that if $f_n \to f$ uniformly on a set S, and if $g_n \to g$ uniformly on S, then $f_n + g_n \to f + g$ uniformly on S.

25. Let $f_n(x) = \frac{x^n}{n}$. Show (f_n) is uniformly convergent on $[-1, 1]$ and specify the limit function.

26. Let $f_n(x) = \frac{n+\cos x}{2n+\sin^2 x}$ for all real numbers x.

    (a) Show (f_n) converges uniformly on $\mathbb{R}$. Hint: First decide what the limit function is; then show (f_n) converges uniformly to it.

    (b) Calculate $\lim_{n\to\infty}\int_2^7 f_n(x)dx$. Hint: Don't integrate f_n.

27. Show $\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$ converges uniformly on $\mathbb{R}$ to a continuous function.

28. Show $\sum_{n=1}^{\infty}\frac{x^n}{n^2 2^n}$ has radius of convergence 2 and the series converges uniformly to a continuous function on $[-2, 2]$.

29. **(a)** Show $\sum\frac{x^n}{1+x^n}$ converges for $x \in [0, 1)$

    (b) Show that the series converges uniformly on $[0, a]$ for each a, $0 < a < 1$.

30. Suppose $\sum_{k=1}^{\infty} g_k$ and $\sum_{k=1}^{\infty} h_k$ converge uniformly on a set S. Show $\sum_{k=1}^{\infty}(g_k + h_k)$ converges uniformly on S.

UNIT-IV

31. Let $f(x) = x$ for rational x and $f(x) = 0$ for irrational x.

    (a) Calculate the upper and lower Darboux integrals for f on the interval $[0, b]$.

    (b) Is f integrable on $[0, b]$?

32. Let f be a bounded function on $[a, b]$. Suppose there exist sequences (U_n) and (L_n) of upper and lower Darboux sums for f such that $\lim(U_n - L_n) = 0$. Show f is integrable and $\int_a^b f = \lim U_n = \lim L_n$.

33. A function f on $[a, b]$ is called a step function if there exists a partition $P = \{a = u_0 < u_1 < ... < u_m = b\}$ of $[a, b]$ such that f is constant on each interval (u_{j-1}, u_j), say $f(x) = c_j$ for x in (u_{j-1}, u_j).

    (a) Show that a step function f is integrable and evaluate $\int_a^b f$.

    (b) Evaluate the integral $\int_0^4 P(x)dx$ for the postage-stamp function.

34. Show $|\int_{-2\pi}^{2\pi} x^2\sin^8(e^x)dx| \le \frac{16\pi^3}{3}$.

35. Let f be a bounded function on$[a, b]$, so that there exists $B > 0$ such that $|f(x)| \leq B$ for all $x \in [a, b]$.

(a) Show

$$U(f^2, P) - L(f^2, P) \leq 2B[U(f, P) - L(f, P)]$$

for all partitions P of $[a, b]$. Hint: $f(x)^2 - f(y)^2 = [f(x) + f(y)].[f(x) - f(y)]$

(b) Show that if f is integrable on $[a, b]$, then f^2 also is integrable on $[a, b]$.

36. Calculate

(a) $\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2} dt$ **(b)** $\lim_{h \to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt$.

37. Show that if f is a continuous real-valued function on $[a, b]$ satisfying $\int_a^b f(x)g(x)dx = 0$ for every continuous function g on $[a, b]$, then $f(x) = 0$ for all x in $[a, b]$.

Skill Enhancement Course – I - FOR ALL SCIENCE FACULTY B.Sc., II YEAR, III Semester
DEPARTMENTS

COMPUTER BASICS AND AUTOMATION

Credits: 2

Theory: 2 hours/week **Marks - 50**

Unit –I BASICS OF COMPUTERS

1.2 Introduction to computers- Computer parts and Characteristics of computer.
1.2.Generations of Computers, Classification of Computers, Basic computer organization.
1.3.Applications of Computer. Input and Output Devices- Input Devices, Output Devices.
1.4. Soft Copy Devices, Hard Copy Devices. Computer Memory and Processors.

Unit – II OFFICE AUTOMATION

1.1.Desktop - Word - Creation of files and folders, recycle Bin.
1.2.Web browser, Office Automation System, need for Office Automation System.
1.3. Excel – Tables, graphs
1.4.PowerPoint, Access to files and folders.

Text Book:

1. Reema Thareja "Fundamentals of Computers" Oxford University Press 2015.

References:

1. A. Goel, Computer Fundamentals, Pearson Education, 2010.
2. Spoken Tutorial on "Linux (Ubuntu), LibreOffice (Writer, Calc, Impress), Firefox", as E-resource for Learning. http://spoken-tutorial.org